# Texture Synthesis via a Non-parametric Markov Random Field

Rupert Paget and Dennis Longstaff

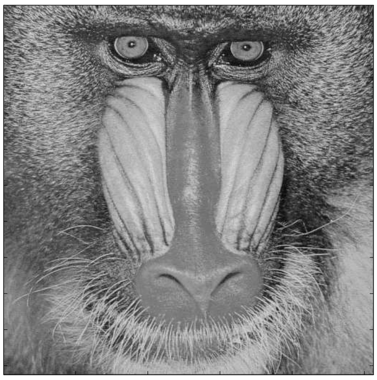
Cooperative Research Centre for Sensor Signal and Information Processing
Dept. Electrical and Computer Engineering
University of Queensland

DICTA'95

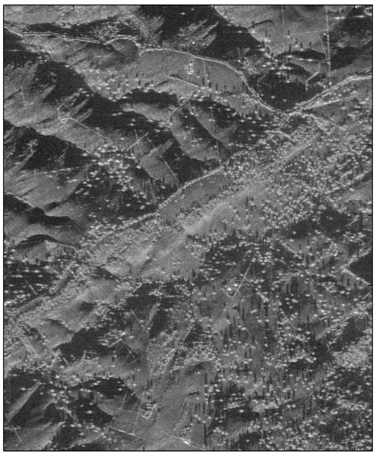
# Introduction

1. What is texture ?
2. Analysis of texture
3. Markov random field (MRF) texture model
4. Non-parametric MRF texture model
5. Multiscale texture synthesis
6. Novel energy function
7. Results

# What is texture ?



Baboon face



Airborne SAR

Figure 1: Texture in Images

**texture** is the visual characteristics within an area of an image that identifies that area as belonging to a certain class.

**class** may be associated with a particular physical interpretation such as

- grass, hair, water, or sand

# Texture analysis

1. why
   - capture the visual characteristics of texture
   - analytically segment and classify the different textures in an image
2. models
   - auto-models → Gaussian MRF model
   - autoregressive (AR) models
   - moving average (MA) models
3. tests
   - segmentation
   - synthesis
4. questions
   - what are the characteristics of texture?
   - how can we identify them?

# Markov Random Field Model

$$\mathcal{P}(x_s|.) = P(x_s|x_r, r \in \mathcal{N}_s) \;\; s \in S \tag{1}$$

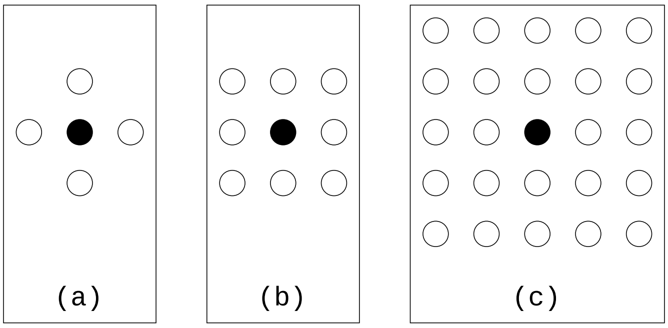



Figure 2: Neighbourhoods. (a) The first order or "nearest-neighbour" neighbourhood ($c = 1$); (b) second order neighbourhood ($c = 2$); (c) eighth order neighbourhood ($c = 8$).

$$\mathcal{N}_s^c = \{r = (k,l) \in S : 0 < (k-i)^2 + (l-j)^2 \leq c\} \tag{2}$$

# Non-parametric MRF Model

$$\mathcal{P}(x_s|.) = P(x_s|x_r, r \in \mathcal{N}_s)\ s \in S \tag{3}$$

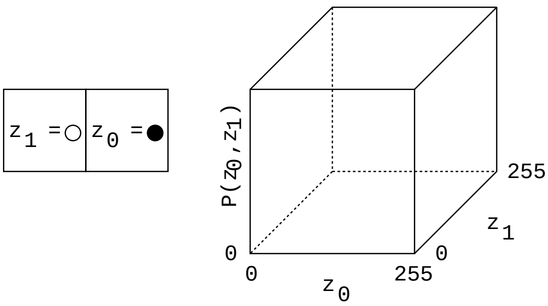



Figure 3: Neighbourhood and histogram

- domain size= (# of grey levels)$^{(\text{\# of neighbours} + 1)}$
- amount of data = size of image

Example

$$\left.\begin{array}{l} 3 \times 3 \text{ neighbourhood} \\ 16 \text{ grey levels} \\ 1000 \times 1000 \text{ image} \end{array}\right\} \Longrightarrow 1 \text{ in } 70000 \tag{4}$$

# Parzen Window Density Estimator

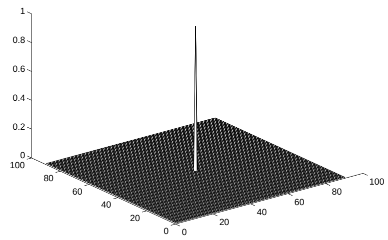
$$\begin{aligned}\mathcal{P}(x_s|.) &= P(x_s|x_r, r \in \mathcal{N}_s) \\ &= \frac{P(x_s, x_r, r \in \mathcal{N}_s)}{P(x_r, r \in \mathcal{N}_s)} \\ &= \frac{\hat{f}(\mathbf{z} = \mathrm{Col}[x_s, x_r, r \in \mathcal{N}_s])}{P(x_r, r \in \mathcal{N}_s)} \\ &= \frac{1}{C_s}\sum_{k=1}^{n} \exp\left(-\frac{1}{2h_{opt}^2}(\mathbf{z}-\mathbf{Z}_k)^{\mathbf{T}}(\mathbf{z}-\mathbf{Z}_k)\right) \qquad (5)\end{aligned}$$

where $C_s$ is a constant with respect to $\{x_r, r \in \mathcal{N}_s\}$

$$C_s = nh_{opt}^d (2\pi)^{d/2} P(x_r, r \in \mathcal{N}_s) \qquad (6)$$

- The sample data $\mathbf{Z}_k = \mathrm{Col}[z_{0_k} = y_{s'}, z_{i_k} = y_{r'} \in \mathcal{N}_{s'}]$
- The variable $\mathbf{z} = \mathrm{Col}[z_0 = x_s, z_i = x_r \in \mathcal{N}_s]$

# Parzen Window Density Estimator




$\Longrightarrow$

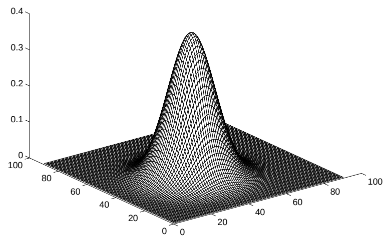



Figure 4: Histogram data convolved with multi-dimensional Gaussian

# Multiscale texture synthesis

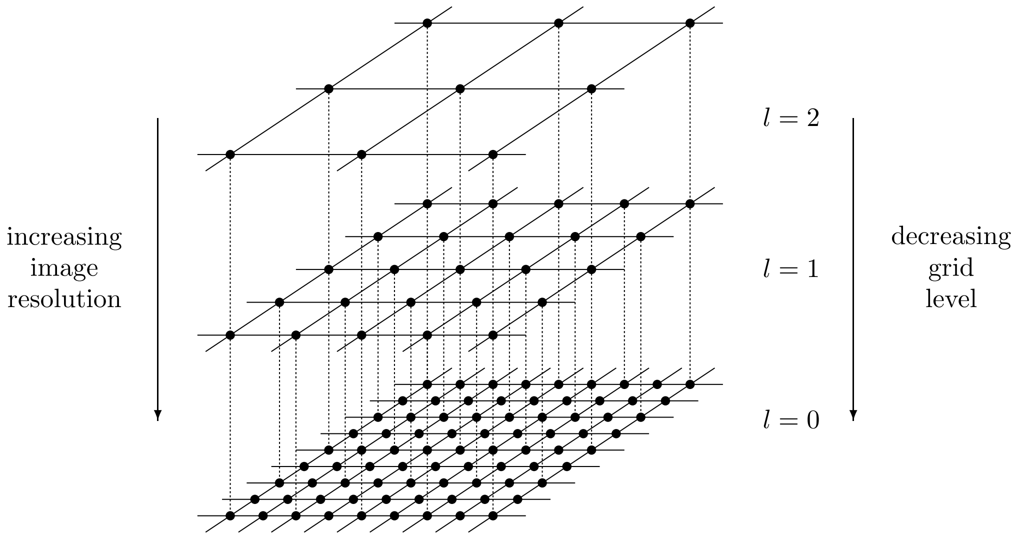



Figure 5: Grid organisation for multiscale modelling of a MRF.

- ICM algorithm with non-parametric $\mathcal{P}(x_s|.)$
- Constraint easy to maintain by decimation

# Energy Function

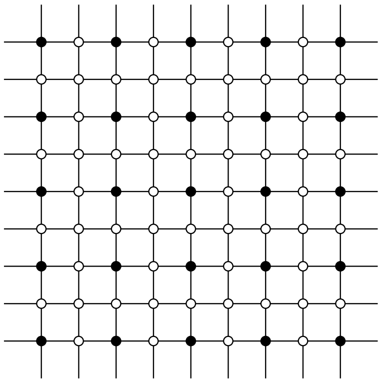


Figure 6: $\bullet$ is a site which is from the above level. $\circ$ is a site which is to undergo relaxation.

Deterministic relaxation at a level

1. initialize $e_s = \begin{cases} 0 & \text{if site} s = \circ \\ 1 & \text{if site} s = \bullet \end{cases}$

2. ICM algorithm

   - Modify Parzen density estimate of $\mathcal{P}(x_s|.)$ from Equation (5) with

$$(\mathbf{z} - \mathbf{Z}_k) = \mathrm{col}[x_s - y_{s'}, (x_r - y_{r-s+s'})e_r, r \in \mathcal{N}_s] \qquad (7)$$

   - Update $e_s$

$$e_s = \frac{1 + \sum_{r \in \mathcal{G}_s} e_r}{|\mathcal{G}_s|} \qquad (8)$$

   where $e_r$ is the energy at the site $r$. If $e_s > 1 \Longrightarrow e_s = 1$.

3. When all $e_s = 1$ at level it becomes time to move down to the next level and begin again.

**Note** $e_s =$ "confidence" in using site $s$ to estimate $\mathcal{P}(x_s|.)$

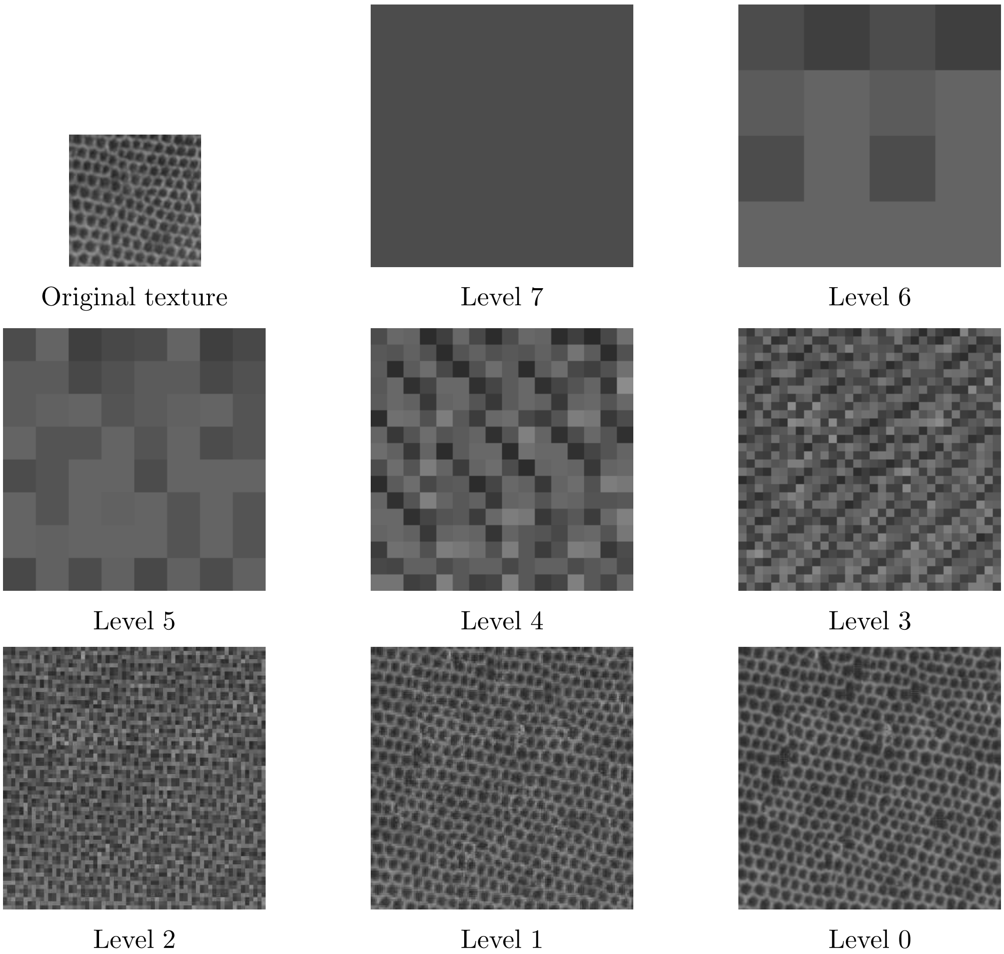



Figure 7: Texture synthesis of Brodatz D22 (reptile skin) with neighbourhood order $c = 8$.

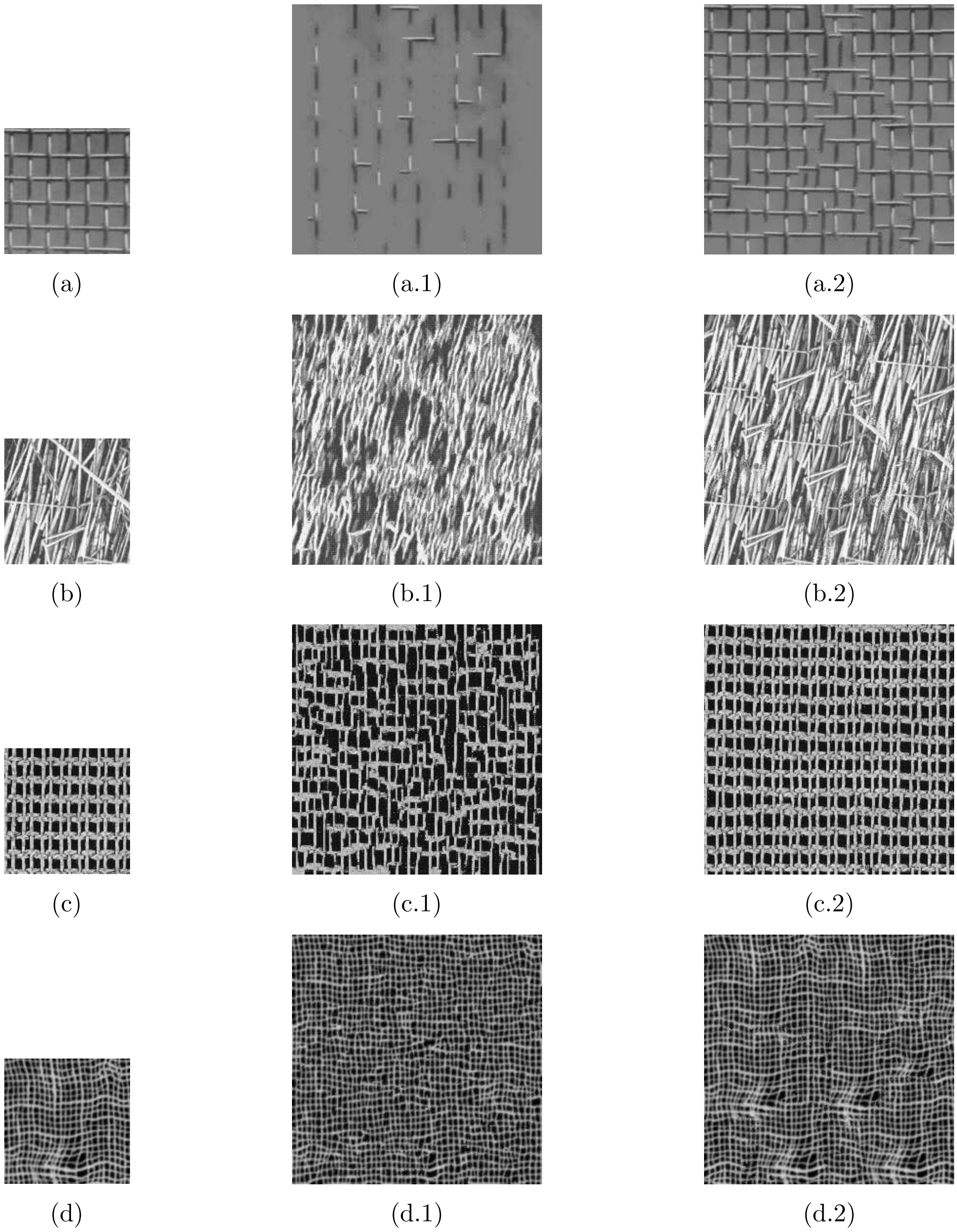



Figure 8: Brodatz textures: (a) D1 - aluminum wire mesh; (b) D15 - straw; (c) D20 - magnified French canvas; (d) D103 - loose burlap; (?.1) textures synthesised using neighbourhood order $c = 8$; (?.2) textures synthesised using neighbourhood order $c = 18$.