A NONPARAMETRIC MULTISCALE MARKOV RANDOM FIELD MODEL FOR SYNTHESISING NATURAL TEXTURES

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ABSTRACT

In this paper we present a non-causal, non-parametric, multiscale, Markov random field (MRF) texture model. The model is capable of capturing the characteristics of and synthesising a wide variety of textures, varying from the highly structured to the stochastic. We introduce a novel multiscale texture synthesis algorithm that allows us to use large neighbourhood systems to model some complex natural textures. As an added advantage of using the novel multiscale texture synthesis algorithm, phase discontinuities in the synthetic textures are reduced. Finally we show how the high dimensional representation of the texture may be modelled with lower dimensional statistics without compromising the integrity of the representation. The power of our modelling technique is evident in that only a small training image is required to derive respectable results even when the texture contains long range characteristics.

1. INTRODUCTION

In an image, texture is the visual characteristics of an image segment that helps identify that segment with a certain physical interpretation, *e.g.,* grass, hair, water or sand. Photographic examples of some natural textures are given in the Brodatz album [3].

In image processing, texture may be defined in terms of spatial interactions between pixel grey levels within a digital image. The aim of texture analysis is to capture the visual characteristics of a texture by mathematically modelling these spatial interactions. If, for a particular texture, these spatial characteristics can be uniquely modelled, it becomes possible to analytically discriminate that texture from other textures. An image may then be segmented into its various textural components with each component being classified according to its model. The difficulty with texture analysis lies in trying to uniquely model the texture.

Usually, when trying to segment an image into its different textural components, all the different possible textural types need to be known. Under such circumstances the texture models only capture enough characteristics to distinguish each texture from all other known textures. This approach is adequate if the images undergoing texture segmentation and classification are similar to the images that were used to train the texture models. However, for images where not all the textural types are known, a texture model needs to capture <u>all</u> the relevant characteristics that uniquely identify a texture. Then, when an image is segmented and classified, the model is used to identify the probability that a particular image segment is of the same texture as the model. This type of texture segmentation and classification is required where not all the textural types are known.

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Unfortunately, with the present knowledge of texture, obtaining a model that captures <u>all</u> the relevant characteristics unique to a particular texture is an open problem. Texture is not fully understood and therefore what constitutes the relevant characteristics are unknown. A reasonable way to test whether a texture model has captured all the relevant characteristics is to synthesise a texture from the model and make a subjective judgement on how similar the synthetic and original texture are.

Current texture models like the auto-models, autoregressive (AR) models, moving average (MA) models, and autoregressive moving average (ARMA) models, have not been able to realistically synthesise natural textures [9] as found in the Brodatz album. In this paper we present a non-parametric, multiscale, Markov random field model that is capable of synthesising natural textures. The synthesis algorithm employed is a multiscale texture synthesis algorithm [8] with a novel pixel temperature function.

Unfortunately, although the synthesis test indicates whether the relevant characteristics have been captured, it does not determine whether the model would be suitable for segmentation and classification. Although the model may contain the relevant characteristics it may also contain superfluous characteristics that would not allow the model to segment and classify similar textures. In this paper we also present a method for reducing the non-parametric multiscale Markov random field model to a set of clique probability functions associated with Markov random field model. In doing so we produce a model that still contains the relevant characteristics but not the superfluous characteristics. An added advantage of this technique is that it helps obtain a better understanding of the type of parametric Markov random field models required for modelling natural textures.

2. TEXTURE MODEL

Markov random field (MRF) models have been used in image restoration, region segmentation, and texture synthesis [4]. The property of a MRF is that: a variable X_s on a lattice $S = \{s = (i,j) : 0 \leq i,j < N\}$ may have its value x_s set to any value, but the probability of $X_s = x_s$ is conditional upon the values x_r at its *neighbouring* sites $r \in \mathcal{N}_s$. A local conditional probability density function (LCPDF) defined over these neighbouring sites $r \in \mathcal{N}_s$ determines the probability of $X_s = x_s$. Therefore the LCPDF,

$$P(X_s = x_s | X_r = x_r, r \in \mathcal{N}_s) \quad s \in S, \tag{1}$$

defines the MRF [1].

To model an image as a MRF, consider each pixel in the image as a site on a lattice, and the grey scale value of that pixel as the value of that site. If the image is all of one texture, then the LCPDF derived from the image is the model that defines the texture.

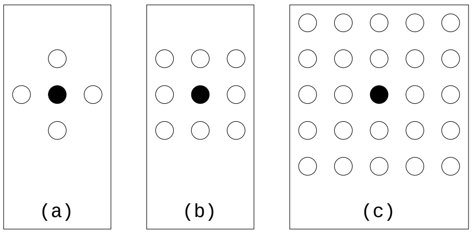


Figure 1: Neighbourhoods. (a) The first order neighbourhood ($c = 1$) or "nearest-neighbour" neighbourhood for the site $s = (i,j) =$ '•' and $r = (k,l) \in \mathcal{N}_s =$ '∘'; (b) second order neighbourhood ($c = 2$); (c) eighth order neighbourhood ($c = 8$).

The neighbourhoods $\mathcal{N}_s^c$ employed in this paper are the same as in [6, 5] defined by,

$$\mathcal{N}_s^c = \{r = (k,l) \in S : 0 < (k-i)^2 + (l-j)^2 \leq c\}, \tag{2}$$

where c refers to the order of the neighbourhood system. Neighbourhood systems for $c = 1, 2$ and 8 are shown in Fig. 1 (a), (b), and (c) respectively.

2.1. Model 1: Non-parametric MRF Model

Given an image of a homogeneous texture, and a predefined neighbourhood system, a non-parametric estimate of the LCPDF may be obtained by building a multi-dimensional histogram.

For example, choose a neighbourhood $\mathcal{N}_s = \{s-1\}$ as shown in Fig. 2(a). To estimate the respective LCPDF, build a 2-dimensional histogram with dimensions (L_0, L_1), where L_0 represents the pixel value x_s and L_1 represents the relative neighbouring pixel value x_{s-1}. Initialize $F(L_0, L_1) = 0 \;\forall\; L_0, L_1$. Then by raster scanning the image, increment the variable $F(L_0 = x_s, L_1 = x_{s-1})$ for each site $s \in S$, $\mathcal{N}_s \subset S$. The simple estimate of the LCPDF is then given by,

$$\hat{P}(x_s | x_{s-1}) = \frac{F(L_0 = x_s, L_1 = x_{s-1})}{\sum_{L_0 \in \Lambda} F(L_0, L_1 = x_{s-1})}. \tag{3}$$

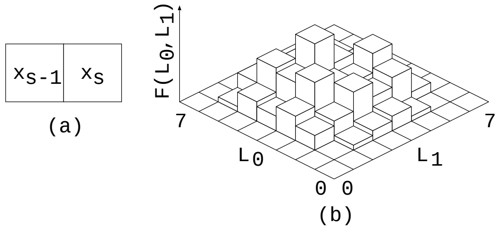


Figure 2: Neighbourhood and its 2-D histogram

The data obtained from the image to build the multi-dimensional histogram is not independent and identically distributed (i.i.d.). However the *pseudo-likelihood estimate* [2] of the LCPDF uses the same non i.i.d. data. Geman and Graffigne [7] proved that the pseudo-likelihood estimate converged to the true LCPDF with a probability one as the image size increased to infinity. With this evidence we justify our use of non i.i.d. data for estimating our non-parametric version of the LCPDF.

The true LCPDF is given by a histogram built from an infinite amount of sample data. Therefore the true LCPDF needs to be estimated from the multi-dimensional histogram. A non-parametric density estimator is advantageous for this type of problem where the domain is only sparsely populated with sample data [11].

2.2. Parzen Window Density Estimator

The Parzen-window density estimator [11] has the effect of smoothing each sample data point in the multi-dimensional histogram over a larger area.

Denote the sample data as $\mathbf{Z}_s = \text{Col}[x_s, x_r, r \in \mathcal{N}_s]$ $s \in S, \mathcal{N}_s \subset S$. For a column vector $\mathbf{z} = \text{Col}[L_0, L_{n_r}, r \in \mathcal{N}_s]$, the Parzen-window density estimated frequency $\hat{F}(\mathbf{z})$ of the previously frequency F is given by [11, p 76] as,

$$\hat{F}(\mathbf{z}) = \frac{1}{nh^d} \sum_{s \in S, \mathcal{N}_s \subset S} K\left\{\frac{1}{h}(\mathbf{z} - \mathbf{Z}_s)\right\}, \tag{4}$$

where n is the number of sample data $\mathbf{Z}_s$, h is the window parameter, and $d = |\mathcal{N}_s| + 1$ equals the number of elements in the vector $\mathbf{z}$.

The shape of the smoothing is defined by the kernel function K. We choose K as the standard multi-dimensional Gaussian density function,

$$K(\mathbf{z}) = \frac{1}{(2\pi)^{d/2}} \exp(-\frac{1}{2}\mathbf{z}^{\mathbf{T}}\mathbf{z}). \tag{5}$$

The size of the kernel function K is defined by the window parameter h. The aim is to correctly choose h so as to obtain the best estimate of the frequency distribution $\hat{F}$ for the LCPDF. From Silverman [11, p 85] an optimal window parameter h_{opt} is,

$$h_{opt} = \sigma \left\{\frac{4}{n(2d+1)}\right\}^{1/(d+4)}, \tag{6}$$

where σ^2 is the the average marginal variance. In our case the marginal variance is the same in each dimension and therefore σ^2 equals the variance associate with the one-dimensional histogram.

2.3. Model 2: Strong Non-parametric MRF Model

The underlying problem with determining the LCPDF is that the domain over which the estimation process is performed is very large and only sparsely populated with sample data. This makes a reasonable estimation of the LCPDF very hard to achieve. The second model presented here has been constructed so as to reduce the domain over which the estimation process is performed, while maintaining the integrity of the LCPDF.

The LCPDF for a strong MRF [10] may be written in terms of its marginal distributions defined over the *cliques* of its neighbourhood. Cliques are subsets of the neighbourhood, where each site in a clique is a neighbour to each other site within the clique [6].

If we denote $P(X_s = x_s | X_r = x_r, r \in C)$ as the conditional probability defined over a clique $C \subset \mathcal{N}_s$, then we find that the LCPDF may be approximated by,

$$P(X_s = x_s | X_r = x_r, r \in \mathcal{N}_s) = \prod_{\substack{C \subset \mathcal{N}_s, \\ C \not\subset C' \subset \mathcal{N}_s}} P(X_s = x_s | X_r = x_r, r \in C), \qquad (7)$$

where each clique probability $P(X_s = x_s | X_r = x_r, r \in C)$ is estimated in the same way as for the previous neighbourhood probability $P(X_s = x_s | X_r = x_r, r \in \mathcal{N}_s)$.

This simple estimate (7) only incorporates those clique probabilities defined on the major cliques of the neighbourhood which are not subsets of other cliques.

3. TEXTURE SYNTHESIS

Texture synthesis is one means by which to test whether the LCPDF has captured the required textural characteristics to model a particular texture. Proof that the model has indeed captured these textural characteristics is obtained by using the model to synthesis representative examples of the same texture.

A texture may be synthesised from a MRF model via a method known as stochastic relaxation (SR). An image may be generated via SR by starting with any image and iteratively updating pixels in the image with respect to the LCPDF. We use the well known SR algorithm, the Gibbs sampler [6].

3.1. Multiscale Relaxation

The basic concept of multiscale relaxation (MR) of an MRF is to relax the image at various "resolutions." The advantage of this is that some texture characteristics are better resolved at some resolutions than at others [8].

The multiscale model may be best described through the use of a multigrid representation of the image as shown in Fig. 3. The grid at level $l = 0$ represents the original image, where each intersection point '•' is a site $s \in S$. The MR algorithm starts at the lowest resolution with a SR algorithm. Once an equilibrium state has been reached, the image is propagated down to the the next level where it undergoes further relaxation. In our algorithm we also use a pixel temperature function that allows the image propagated down from the previous level to be better incorporated into the next relaxation process.

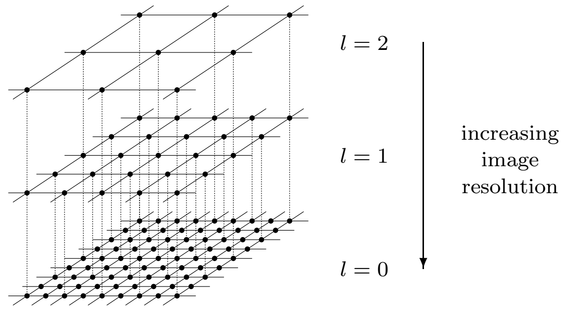


Figure 3: Grid organisation for multiscale modelling via decimation. Only connections representing nearest-neighbour interactions are included.

3.2. Pixel Temperature Function

The object of the pixel temperature function is to define a degree of "confidence" in a pixel having the correct value. Every pixel has its own individual temperature t_s representing the confidence associated with the pixel x_s. The confidence is expressed as a value $0 \leq t_s \leq 1$, where 1 represents complete confidence and 0 none at all.

The pixel temperature is incorporated into the LCPDF by modifying the form of $(\mathbf{z} - \mathbf{Z}_s)$ in (4) to,

$$(\mathbf{z} - \mathbf{Z}_s) = \mathrm{col}[L_0 - x_s, (L_{n_r} - x_r)t_r, r \in \mathcal{N}_s]. \qquad (8)$$

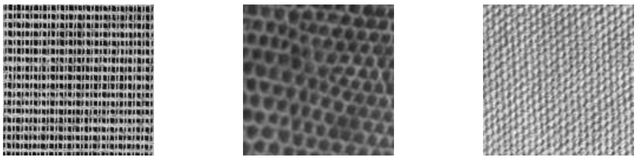
Before the SR algorithm starts at level l, those pixels which have been relaxed at the previous level $l + 1$ are given a pixel temperature $t_s = 1$ and therefore complete confidence. The other pixels have their pixel temperatures initialized to $t_s = 0$ and therefore no confidence. After each iteration of the SR algorithm, those pixels that were relaxed have their temperatures updated. We chose the following formula to describe the updated confidence,

$$t_s = \frac{1 + \sum_{r \in \mathcal{N}_s} t_r}{|\mathcal{N}_s|}. \qquad (9)$$

Initially, only those sites that have had their values relaxed at the previous grid level are used in the LCPDF. However, as the SR iterations progress, more sites gain a degree of confidence. When $t_s = 1 \; \forall \; s \in S$ the image is propagated to the next lower grid level where the relaxation process begins again.

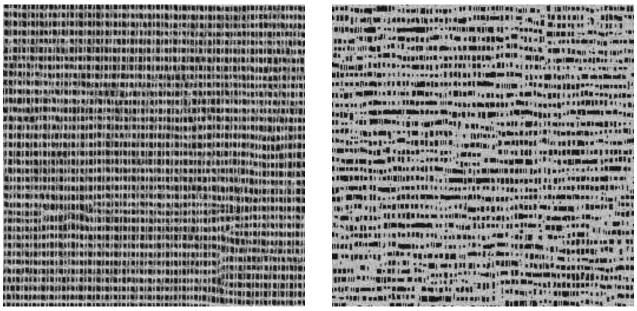
4. RESULTS

A 128×128 image was used to derive the texture model, from which a 256×256 image was synthesised. This was done in order to confirm that the characteristics of the texture had indeed been captured by the model.



(a) (b) (c)

Figure 4: Original Brodatz textures: (a) Brodatz D21 (French canvas); (b) Brodatz D22 (Reptile skin); (c) Brodatz D77 (Cotton canvas).



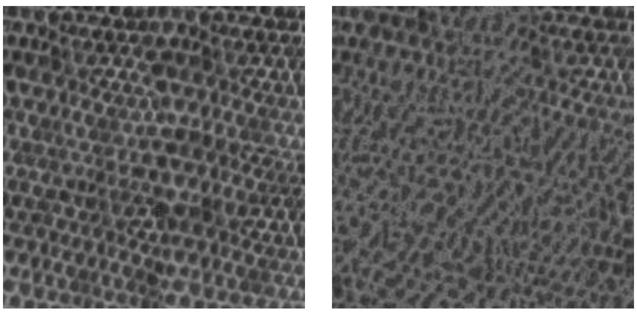
(a) (b)

Figure 5: Synthesised Brodatz D21 textures: (a) Model 1 (neighbourhood order c=18); (b) Model 2 (neighbourhood order c=8 with 3rd order cliques).

5. CONCLUSION

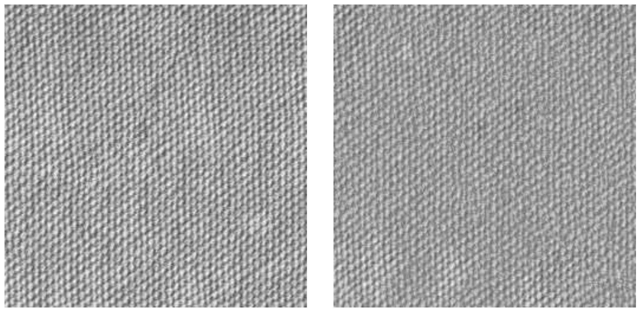
From the results shown in Figs. 5–7, we believe that the non-causal non-parametric multiscale Markov random field texture model forms a highly representative model of the texture.

The results obtained for second model, shown in Fig. 5(b), Fig. 6(b), and Fig. 7(b), suggests that these textures may be successively modelled with just third order statistics. Third order statistics have the advantage of being easier to use in a segmentation or classification algorithm as compared to a complicated function defined over a multi-dimensional histogram, as was used in the first model.



(a) (b)

Figure 6: Synthesised Brodatz D22 textures: (a) Model 1 (neighbourhood order c=18); (b) Model 2 (neighbourhood order c=8 with 3rd order cliques).



(a) (b)

Figure 7: Synthesised Brodatz D77 textures: (a) Model 1 (neighbourhood order c=18); (b) Model 2 (neighbourhood order c=8 with 3rd order cliques).

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